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Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, February 10, 2015

Prof. Erich Kaltofen <kaltofen@math.ncsu.edu>
www.math.ncsu.edu/~kaltofen/courses/NumberTheory/Spring15/ (URL)

919.515.8785 (phone)
919.515.3798 (fax)

Your Name: ______________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 8 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
5	_____
Total	_____

Problem 1 (18 points)

(a, 5pts) Please give the complete integral solution (with an integer parameter λ) for the diophantine linear equation $2u+15z=10$ in the integer variables u and z. Please show your work.

(b, 5pts) Please give the complete integral solution (with the integer parameters u, μ) for the diophantine linear equation $6x+10y=2u$ in the integer variables x and y.
Then substitute the solution from Part (a) for u, z to obtain the complete integer solution for the diophantine linear equation $6x+10y+15z=10$ in the integer variables x, y, z. Please show your work.

(c, 4pts) Please write the rational number $\frac{90}{55}$ as a (finite) continued fraction $q_1+\cfrac{1}{q_2+\cfrac{1}{q_3+\cfrac{1}{\ddots}}}$.

(d, 4pts) Please list all prime numbers p with $1010 \leq p \leq 1020$. You can make use of the fact that $\pi(1010)=169$ and $\pi(1020)=171$. Please show your work.

Problem 2 (8 points): Please prove for all integers $a \in \mathbb{Z}$: $\mathrm{GCD}(11a-12, 7a-8, 2a+1) = 1$.

Problem 3 (8 points): Let $k \in \mathbb{Z}_{\geq 0}$ be fixed. Please prove for all integers $n \in \mathbb{Z}_{\geq 0}$ the following identity for binomial coefficients:

$$\sum_{i=0}^{n} \binom{k+i}{k} = \binom{k}{k} + \binom{k+1}{k} + \cdots + \binom{k+n}{k} = \binom{k+n+1}{k+1}.$$

Problem 4 (5 points): Please state **three** proven theorems on the prime numbers.

Problem 5 (5 points): Please state **three** unproven conjectures on the prime numbers.