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Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, February 13, 2012

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Your Name: ______________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 8 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	____
2	____
3	____
4	____
5	____
Total	____

Problem 1 (18 points)

(a, 5pts) Please give the solution (with an integer parameter λ) for the diophantine equation $315x+217y=14$ in the integer variables x and y. Please show your work.

(b, 5pts) Please consider the expansion of the trinomial $(x+y+z)^6$. What is the multinomial coefficient of the term $x^2y^2z^2$ in that expansion?

(c, 4pts) Please list 5 prime numbers of the form n^2+1, where $n \in \mathbb{Z}_{\geq 1}$.

(d, 4pts) From the fact that $p_{55}=257$, where p_{55} is the 55-th prime number and $p_1=2, p_2=3, p_3=5$, $\ldots$, deduce the value of $\pi(262)$. Please show your work.

Problem 2 (8 points): Please prove for all integers $a, b \in \mathbb{Z}$: $\mathrm{GCD}(a+2b-1, 2a+b+2, b-1) = 1$. [Hint: find Bezout coefficients; you can eliminate b from the first and second argument using the third.]

Problem 3 (8 points): Please prove for all integers $n \in \mathbb{Z}_{\geq 2}$:

$$\sum_{i=1}^{n-1} \frac{1}{i(i+1)} = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{(n-1)n} = 1 - \frac{1}{n}.$$

Problem 4 (5 points): True or false: for all p that are prime numbers and for all $i \in \mathbb{Z}$ with $1 \leq i \leq p-1$ the binomial coefficient $\binom{p}{i}$ is divisible by p. Please explain. [Hint: consider the factorial representation of the binomial coefficient.]

Problem 5 (5 points): True or false: there are infinitely many prime numbers whose decimal representation ends with 001, i.e., are of the form $1000k+1$ for integers k. Please explain.