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Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, Feb 15, 2010

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Your Name: ____________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 9 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **75 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
5	_____
Total	_____

Problem 1 (18 points)

(a, 4pts) Please give the solution (with an integer parameter μ) in the variables z and u for the diophantine equation $5u + 21z = 2$.

(b, 5pts) Please give the parametric solution in the variables x and y for the diophantine equation $35x + 15y = 5u$. Your solution has the right-side integer multiplier u and an additional parameter λ.

(c, 5pts) Please give a parametric solution (with integer parameters λ and μ) in the variables x, y and z for the diophantine equation $35x + 15y + 21z = 2$.

(d, 4pts) Please compute $\pi(58) - \pi(30)$. Please show your work.

Problem 2 (8 points): Consider the expansion of the trinomial power $(x+y+z)^6 = x^6 + 6x^5y + 6x^5z + \cdots$.

(a, 4pts) What is the coefficient of the term x^2y^3z in the expansion?

(b, 4pts) How many terms $x^iy^jz^k$ where $i+j+k=6$ occur in the expansion? Hint: think of selecting 6 times from 3 objects x, y, z.

Problem 3 (8 points): Consider the sequence a_n that is inductively defined for all integers $n \geq 0$ by $a_0 = 1$, $a_1 = 0$ and $a_{n+2} = -a_{n+1} + a_n$. Thus the next elements are $a_2 = 1$, $a_3 = -1$, $a_4 = 2, \ldots$ Please prove that $a_n = (-1)^n f_{n-2}$ for all integers $n \geq 2$, where f_n are the Fibonacci numbers: $f_0 = 1$ (note that the textbook differently initializes the 0th Fibonacci number to 0), $f_1 = 1$ and $f_{n+2} = f_{n+1} + f_n$ for all $n \geq 0$.

Problem 4 (5 points): True or false: The set $\{9k+4 \mid k \in \mathbb{Z}_{\geq 0}\}$ contains infinitely many prime numbers. Please explain.

Problem 5 (5 points): Please state the Fundamental Theorem of Arithmetic.