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Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, Feb 11, 2009

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Your Name: ____________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 8 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **60 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
5	_____
Total	_____

Problem 1 (18 points)

(a, 5pts) Please compute $g = \gcd(224, 156)$ and $s, t \in \mathbb{Z}$ such that $224s + 156t = g$. Please show all work.

(b, 4pts) Please give a parametric solution for the diophantine equation $224x + 156y = 8$.

(c, 4pts) Please compute $\pi(105)$. Please show your work.

(d, 5pts) Alice and Bob are choosing from 6 objects, A, B, C, D, E, and F. First, Alice chooses 2 and then Bob chooses 3 from the rest. How many different results are possible?

Problem 2 (8 points): Please prove for all integers $n \geq 1$: $\frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n-1}$.

Problem 3 (8 points): Consider the sequence a_n that is inductively defined for all integers $n \geq 0$ by $a_0 = 0$, $a_1 = 2$ and $a_{n+2} = 4a_{n+1} - 4a_n$. Thus the next elements are $a_2 = 8$, $a_3 = 24$, $a_4 = 64, \ldots$ Please prove that $a_n = n \cdot 2^n$ for all integers $n \geq 0$.

Problem 4 (5 points): Please find all twin prime pairs (F_n, M_p), where F_n is a Fermat prime and M_p is a Mersenne prime.

Problem 5 (5 points): Please state the Green-Tao theorem on primes in an arithmetic progression.