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Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, Feb 13, 2008

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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 9 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **60 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

Total _____

Problem 1 (18 points)

(a, 5pts) Please compute $g = \gcd(732, 213)$ and $s, t \in \mathbb{Z}$ such that $732s + 213t = g$. Please show all work.

(b, 4pts) Please give the prime factorization of $\binom{13}{7}$.

(c, 4pts) Please list all prime numbers ≤ 100.

(d, 5pts) You are choosing 3 times from 4 objects, *A*, *B*, *C* and *D*. How many combinations with repetition are possible?

Problem 2 (8 points): Please prove that $\gcd(9a-4, 5a-2, 2a+1) = 1$ for all $a \in \mathbb{Z}$.

Problem 3 (8 points): Consider the sequence of Fibonacci numbers f_n that is inductively defined for all integers $n \geq 0$ by $f_0 = 1$, $f_1 = 1$ and $f_{n+2} = f_{n+1} + f_n$. Please prove by induction that $f_n \leq \left(\frac{1+\sqrt{5}}{2}\right)^n$ for all integers $n \geq 0$.

Problem 4 (5 points): Of which we know more in count: Mersenne primes or Fermat primes?

Problem 5 (5 points): Please state Dirichlet's theorem covered in class.