North Carolina State University is a land-grant university and a constituent institution of The University of North Carolina

Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, Feb 14, 2007

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Your Name: _____

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 9 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **60 minutes** to do this test.

Good luck!

Problem 1 _____

2 _____

3 _____

4 _____

5 _____

Total _____

Problem 1 (18 points)

(a, 5pts) Please compute $g = \gcd(325, 120)$ and $s, t \in \mathbb{Z}$ such that $325s + 120t = g$. Please show all work.

(b, 5pts) Please give the prime factorization of $\binom{12}{6}$.

(c, 4pts) Knowing that $p_{50} = 229$, (the 50-th prime number) please compute $\pi(232)$.

(d, 4pts) You are choosing 10 times from 2 objects, *A* and *B*. How many combinations with repetition are possible?

Problem 2 (8 points): Please prove for all integers $n \geq 1$ that $\sum_{i=1}^{n}(2i-1) = n^2$.

Problem 3 (8 points): Please prove by induction on n that for all integers $n \geq 1$, $3^{3n+1} + 2^{n+1}$ is divisible by 5.

Problem 4 (5 points): Please list the first 3 Mersenne primes and the first 3 Fermat primes.

Problem 5 (5 points): Please state the prime number theorem.