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Department of Mathematics

NC STATE UNIVERSITY

MA 410 Theory of Numbers, first mid-semester examination, Feb 14, 2005

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Your Name: _________________________

For purpose of anonymous grading, please do **not** write your name on the subsequent pages.

This examination consists of 5 problems, which are subdivided into 9 questions, where each question counts for the explicitly given number of points, adding to a total of **44 points**. Please write your answers in the spaces indicated, or below the questions, using the **back of the sheets** for completing the answers and **for all scratch work,** if necessary. You are allowed to consult **one** 8.5in $\times$ 11in sheet with notes, but **not** your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have **60 minutes** to do this test.

Good luck!

Problem 1	_____
2	_____
3	_____
4	_____
5	_____
Total	_____

Problem 1 (18 points)

(a, 5pts) Please compute $g = \gcd(272, 119)$ and $s, t \in \mathbb{Z}$ such that $272s + 119t = g$. Please show all work.

(b, 4pts) Please compute the binomial coefficient $\binom{9}{4}$. Please show all work.

(c, 5pts) Please compute p_{20} (the 20-th prime number) and $\pi(50)$.

(d, 4pts) Please give the prime factorization of 14!.

Problem 2 (8 points): Please prove that $\gcd(17a+12, 3a+2) = 2$ for all even integers a, and that $\gcd(17a+12, 3a+2) = 1$ for all odd integers a.

Problem 3 (8 points): Consider the sequence a_n that is inductively defined for all integers $n \geq 0$ by $a_0 = 2$, $a_1 = 1$ and $a_{n+2} = a_{n+1} + 2 \cdot a_n$. Thus the next elements are $a_2 = 5$, $a_3 = 7$, $a_4 = 17$, $a_5 = 31$, $a_6 = 65, \ldots$ Please prove that $a_n = 2^n + (-1)^n$ for all integers $n \geq 0$.

Problem 4 (5 points): True or false: in $\mathbb{Z}_{>0}$ there exist 2^{100} **consecutive** composite integers. Please explain.

Problem 5 (5 points): True or false: Between 2^{100} and 2^{101} there is at least one prime number. Please explain.