```
> restart;# ?Groebner
> F := [x*y-x^2*y^2, y^3-x];
```

$$F := [-x^2 y^2 + x y, y^3 - x] \tag{1}$$

```
> with(Groebner);
```

[*Basis*, *FGLM*, *HilbertDimension*, *HilbertPolynomial*, *HilbertSeries*, *Homogenize*, *InitialForm*, *InterReduce*, *IsBasis*, *IsProper*, *IsZeroDimensional*, *LeadingCoefficient*, *LeadingMonomial*, *LeadingTerm*, *MatrixOrder*, *MaximalIndependentSet*, *MonomialOrder*, *MultiplicationMatrix*, *MultivariateCyclicVector*, *NormalForm*, *NormalSet*, *RationalUnivariateRepresentation*, *Reduce*, *RememberBasis*, *SPolynomial*, *Solve*, *SuggestVariableOrder*, *Support*, *TestOrder*, *ToricIdealBasis*, *TrailingTerm*, *UnivariatePolynomial*, *Walk*, *WeightedDegree*] (2)

```
> G := Basis(F, tdeg(x,y));
```

$$G := [y^3 - x, x^3 - x y^2, x^2 y^2 - x y] \tag{3}$$

```
> h := randpoly([x,y],coeffs=rand(-5..5),degree=4);
```

$$h := 5 x^2 y^2 - 5 y^4 - y^3 - 2 y^2 + 5 x \tag{4}$$

```
> NormalForm(h,G,tdeg(x,y));
```

$$-2 y^2 + 4 x \tag{5}$$

Create a polynomial that is congruent h modulo (F)

```
> h2 := expand(h + randpoly([x,y])*F[1] + randpoly([x,y])*F[2]);
```

$$\begin{aligned} h2 := {} & 40 x^5 y^4 - 42 x^3 y^6 + 7 x^6 y^2 + 98 x^3 y^5 - 30 x^4 y^3 + 87 x^3 y^4 + 42 x^2 y^5 + 44 x y^6 + 29 y^7 \\ & - 7 x^5 y - 98 x^4 y^2 + 75 x^3 y^3 + 37 x^2 y^4 - 23 y^6 - 87 x^4 y + 7 x^3 y^2 - 44 x^2 y^3 - 29 x y^4 \\ & - 37 x^3 y - 70 x^2 y^2 + 23 x y^3 - 5 y^4 - 17 x^2 y - y^3 - 2 y^2 + 5 x \end{aligned} \tag{6}$$

The remainder with respect to the GB of F is the same

```
> NormalForm(h2,G,tdeg(x,y));
```

$$-2 y^2 + 4 x \tag{7}$$

```
> Basis(F, tdeg(y,x));
```

$$[-x^3 + x y^2, y^3 - x, x^4 - x y, x^3 y - x^2] \tag{8}$$

```
> Basis(F, plex(x,y));
```

$$[y^8 - y^4, -y^3 + x] \tag{9}$$

```
> Basis(F, plex(y,x));
```

$$[x^6 - x^2, -x^4 + x y, y^3 - x] \tag{10}$$

```
> F2 := [op(F),x^2+y^2-1];
```

$$F2 := [-x^2 y^2 + x y, y^3 - x, x^2 + y^2 - 1] \tag{11}$$

```
> Basis(F2,tdeg(x,y));
```

$$[1] \tag{12}$$

```
> F3 := map(expand,[randpoly([x,y])*F[1] + randpoly([x,y])*F[2],op
  (F)]);
```

$$F3 := [10x^4y^5 - 15x^4y^4 - 91x^3y^5 - 68x^2y^6 - 40x^5y^2 - 10x^3y^4 + 96x^5y - 81x^3y^3 + 76x^2y^4 + 68xy^5 - 59y^6 + 40x^4y + 47x^2y^3 - 27xy^4 + 15x^3y + 75xy^3 + 30y^4 + 27x^2y - 47xy^2 - 16x^2 - 30xy,\ -x^2y^2 + xy, y^3 - x] \quad (13)$$

```
> Basis(F3,plex(y,x));
```

$$[x^6 - x^2,\ -x^4 + xy, y^3 - x] \quad (14)$$

```
> with(PolynomialTools);
```

$$[AnnihilatingPolynomial, CoefficientList, CoefficientVector, FromCoefficientList, FromCoefficientVector, GcdFreeBasis, GreatestFactorialFactorization, Homogenize, Hurwitz, IsHomogeneous, IsSelfReciprocal, MinimalPolynomial, PDEToPolynomial, PolynomialToPDE, ShiftEquivalent, ShiftlessDecomposition, Shorten, Shorter, Sort, Split, Splits, SquareFreePart, Translate] \quad (15)$$

```
> #with(RandomTools);
> f1 := randpoly([x,y,z],coeffs=rand(-2..2),degree=2);
```

$$f1 := -2x^2 - xy - xz - 2x - 2 \quad (16)$$

```
> f2 := randpoly([x,y,z],coeffs=rand(-2..2),degree=2);
```

$$f2 := -x^2 - 2xy + 2yz - 2z^2 + x - 2y \quad (17)$$

```
> f3 := randpoly([x,y,z],coeffs=rand(-2..2),degree=3);
```

$$f3 := yz^2 - xy + 2yz + 2z^2 - z \quad (18)$$

```
> G_t_xyz := Basis([f1,f2,f3],tdeg(x,y,z)):
  for i from 1 to nops(G_t_xyz) do print(indets(G_t_xyz[i]),G_t_xyz
  [i]); od;
```

$$\{x, y, z\}, 3xy - xz - 4yz + 4z^2 - 4x + 4y - 2$$

$$\{x, y, z\}, 3x^2 + 2xz + 2yz - 2z^2 + 5x - 2y + 4$$

$$\{x, y, z\}, 3yz^2 - xz + 2yz + 10z^2 - 4x + 4y - 3z - 2$$

$$\{x, y, z\}, -12xz^2 + 9y^2z - 33z^3 + 26xz - 9y^2 - 55yz - 74z^2 + 17x - 8y + 36z - 5$$

$$\{x, y, z\}, 75xz^2 + 27y^3 + 267z^3 - 239xz + 18y^2 + 526yz + 863z^2 - 212x + 185y - 360z + 2$$

$$\{x, y, z\}, 27z^4 - 69xz^2 + 111z^3 - 52xz + 9y^2 + 2yz - 92z^2 - x - 14y - 5$$

$$\{x, y, z\}, 27xz^3 + 174xz^2 - 6z^3 + 50xz + 36y^2 + 50yz - 122z^2 + 146x - 152y + 54z + 136 \quad (19)$$

```
> G_p_xyz:=Basis([f1,f2,f3],plex(x,y,z)):
  for i from 1 to nops(G_p_xyz) do print(indets(G_p_xyz[i]),G_p_xyz
  [i]); od;
```

$$\{z\},\ 27z^{10}+276z^9+662z^8+154z^7+1609z^6-463z^5+1198z^4+50z^3+390z^2+24$$

$$\{y,z\},\ -1118394555417z^9-11588875815390z^8-28860989323502z^7-8447422484346z^6-62364236843143z^5+12380884112151z^4-42478016505916z^3-5636981207550z^2+3148284323712y-11206098253566z-1624053725148$$

$$\{x,z\},\ -179167195269z^9-1832984779686z^8-4383653194454z^7-829519025202z^6-10301315596059z^5+2667465809739z^4-6297019037036z^3-2031961376038z^2+524714053952x-1498241271974z+52088085844 \quad (20)$$

```
> factor(G_p_xyz[1]);
```

$$27z^{10}+276z^9+662z^8+154z^7+1609z^6-463z^5+1198z^4+50z^3+390z^2+24 \quad (21)$$

```
> G_p_zxy:=Basis([f1,f2,f3],plex(z,x,y)):
  for i from 1 to nops(G_p_zxy) do print(indets(G_p_zxy[i]),G_p_zxy
  [i]); od;
```

$$\{y\},\ 27y^{10}+51y^9+79y^8+2913y^7+3032y^6-10069y^5+16058y^4-14311y^3+8298y^2-3456y+750$$

$$\{x,y\},\ -8732235346770069y^9-21564495707224191y^8-33922333482811450y^7-962779107921362758y^6-1509286389523468689y^5+2731040685976822144y^4-3727272263882769310y^3+2274057133617108306y^2+130552911299532537x-989222907996925944y+358110453416352411$$

$$\{y,z\},\ 498226451062491y^9+3055727844002235y^8+7124057370031550y^7+6675884415488974 7y^6+2858084289572440 47y^5+2624619330222426 94y^4-1758103816368110 5y^3+1115856602777197 38y^2+1190109372156970 77y+130552911299532537z-85131470025517956 \quad (22)$$

```
> g1 := expand((x-1)^2*(x-2)^2*(x-3)^2); g2 := expand((x-1)^2*(x-2)
  ^3);
```

$$g1 := x^6-12x^5+58x^4-144x^3+193x^2-132x+36$$

$$g2 := x^5-8x^4+25x^3-38x^2+28x-8 \quad (23)$$

```
> GcdFreeBasis([g1,g2]);
```

$$[x-2,\ x^2-6x+9,\ x^2-2x+1] \quad (24)$$

```
> map(factor,%);
```

$$[x-2,\ (x-3)^2,\ (x-1)^2] \quad (25)$$

```
> #?randpoly
>
```