```
> restart;
> with(NumberTheory);
```

[*AreCoprime*, *CalkinWilfSequence*, *CarmichaelLambda*, *ChineseRemainder*, *ContinuedFraction*, *ContinuedFractionPolynomial*, *CyclotomicPolynomial*, *Divisors*, *FactorNormEuclidean*, *HomogeneousDiophantine*, *ImaginaryUnit*, *InhomogeneousDiophantine*, *IntegralBasis*, *InverseTotient*, *IsCyclotomicPolynomial*, *IsMersenne*, *IsSquareFree*, *IthFermat*, *IthMersenne*, *JacobiSymbol*, *KroneckerSymbol*, *Landau*, *LargestNthPower*, *LegendreSymbol*, *Möbius*, *ModExtendedGCD*, *ModularLog*, *ModularRoot*, *ModularSquareRoot*, *Moebius*, *MultiplicativeOrder*, *Möbius*, *NearestLatticePoint*, *NextSafePrime*, *NumberOfIrreducibleFactors*, *NumberOfPrimeFactors*, Ω, Φ, *PrimeCounting*, *PrimeFactors*, *PrimitiveRoot*, *PseudoPrimitiveRoot*, *QuadraticResidue*, *Radical*, *RepeatingDecimal*, *RootsOfUnity*, *SimplestRational*, *SumOfDivisors*, *SumOfSquares*, *ThueSolve*, *Totient*, λ, μ, ϕ, π, σ, τ, φ] (1)

```
> rn := ContinuedFraction(90/21);
```

$$rn := 4 + \cfrac{1}{3 + \cfrac{1}{2 + 0}} \tag{2}$$

```
> for i from 0 to 2 do Convergent(rn,i); od;
```

$$4$$
$$\frac{13}{3}$$
$$\frac{30}{7} \tag{3}$$

```
> m := 2048; e := 20; f := 50; m_bnd:=(e-1)*(f-1);
  r := -30/7 mod m;
```

$$m := 2048$$
$$e := 20$$
$$f := 50$$
$$m_bnd := 931$$
$$r := 1166 \tag{4}$$

```
> rrn := ContinuedFraction(r/m);
```

$$rrn := 0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{9 + \cfrac{1}{2 + \cfrac{1}{7 + 0}}}}}} \tag{5}$$

```
> found:=false:
  for i from 1 to 7 do
     crrn := Convergent(rrn, i):
     num := numer(crrn): den := denom(crrn):
     print(i,"-th fraction:",crrn):
     if den >= e and not found
```

```
        then print(cand); found:= true;
    fi;
    cand:=(-m*num + r*den)/den;
od:
```

$$1, \text{"-th fraction:"}, 1$$

$$2, \text{"-th fraction:"}, \frac{1}{2}$$

$$3, \text{"-th fraction:"}, \frac{4}{7}$$

$$4, \text{"-th fraction:"}, \frac{37}{65}$$

$$-\frac{30}{7}$$

$$5, \text{"-th fraction:"}, \frac{78}{137}$$

$$6, \text{"-th fraction:"}, \frac{583}{1024}$$

$$7, \text{"-th fraction:"}, \frac{583}{1024} \qquad (6)$$

```
> Digits:=50:
```

```
> x := evalf(Pi, 50);
```

$$x := 3.1415926535897932384626433832795028841971693993751 \qquad (7)$$

```
> y := x-3;
```

$$y := 0.1415926535897932384626433832795028841971693993751 \qquad (8)$$

```
> 1/y;
```

$$7.062513305931045769793005152570558042734310025145 8 \qquad (9)$$

```
> z := %-7;
```

$$z := 0.062513305931045769793005152570558042734310025145 8 \qquad (10)$$

```
> 1/z;
```

$$15.996594406685719888923060404755274605671605031492 \qquad (11)$$

```
> cf_pi:=ContinuedFraction(Pi);
  Term(cf_pi,1..30);
```

$$cf_pi := 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \ldots}}}}}}}}}$$

$$[7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1, 84, 2, 1, 1, 15, 3, 13, 1, 4, 2] \qquad (12)$$

```
> for i from 0 to 10 do cv := Convergent(cf_pi,i);
  print(i,cv, evalf(Pi-cv, 30));
```

```
od:
```

$$0,\ 3,\ 0.14159265358979323846264338328$$

$$1,\ \frac{22}{7},\ -0.00126448926734961868021375958$$

$$2,\ \frac{333}{106},\ 0.00008321962752908751924715686$$

$$3,\ \frac{355}{113},\ -2.6676418906242231236893\ 10^{-7}$$

$$4,\ \frac{103993}{33102},\ 5.7789063439038188851\ 10^{-10}$$

$$5,\ \frac{104348}{33215},\ -3.3162780624607255831\ 10^{-10}$$

$$6,\ \frac{208341}{66317},\ 1.2235653294218859793\ 10^{-10}$$

$$7,\ \frac{312689}{99532},\ -2.914338493485691813\ 10^{-11}$$

$$8,\ \frac{833719}{265381},\ 8.71546725822407670\ 10^{-12}$$

$$9,\ \frac{1146408}{364913},\ -1.61074001989903094\ 10^{-12}$$

$$10,\ \frac{4272943}{1360120},\ 4.0406691895606157\ 10^{-13} \qquad (13)$$

```
> ContinuedFraction(Convergent(cf_pi,5));
```

$$3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{293+0}}}} \qquad (14)$$

```
> ContinuedFraction(exp(1));
```

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\ldots}}}}}}}}} \qquad (15)$$

```
> ContinuedFraction((exp(1)^2 - 1)/(exp(1)^2 + 1));
```

(16)

$$0 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{7 + \cfrac{1}{9 + \cfrac{1}{11 + \cfrac{1}{13 + \cfrac{1}{15 + \cfrac{1}{17 + \ldots}}}}}}}}}} \quad \textbf{(16)}$$

```
> degree(0, 'x');
```

$$-\infty \quad \textbf{(17)}$$

```
> ?ThueSolve
> ?
```

$$`?` \quad \textbf{(18)}$$

```
>
```