RESULTANTS

$$F(x) = f_n x^n + \cdots + f_0 \qquad G(x) = g_m x^m + \cdots + g_0$$

$$R(F,G) = \underbrace{\left\| \begin{array}{cccccccc}
f_n & f_{n-1} & \cdots & f_0 & & & & \\
 & f_n & f_{n-1} & \cdots & f_0 & & & \\
 & & \ddots & & & \ddots & & \\
 & & & f_n & f_{n-1} & f_{n-2} & \cdots & f_0 \\
 & & & & f_n & f_{n-1} & \cdots & f_1 \\
g_m & g_{m-1} & \cdots & g_1 & g_0 & & & \\
 & g_m & g_{m-1} & \cdots & g_1 & g_0 & & \\
 & & \ddots & & & & \ddots & \\
 & & & g_m & g_{m-1} & \cdots & & g_0 \\
 & & & & g_m & \cdots & & g_1
\end{array} \right\|}_{n+m}
\begin{array}{l}
\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} m \\
\left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} n
\end{array}$$

$$x^{n+m-1} f_n + x^{n+m-2} f_{n-1} + \cdots + x^{m-1} f_0 = x^{m-1} F$$

$$x^{n+m-2} f_n + \cdots + x^{m-2} f_0 = x^{m-2} F$$

$$\vdots$$

$$x^{n+1} f_n + \cdots + x f_0 = x F$$

$$x^{n} f_n + \cdots + x f_1 + f_0 = F$$

$$x^{m+n-1} g_m + x^{m+n-2} g_{m-1} + \cdots + x^{n-1} g_0 = x^{n-1} G$$

$$x^{m+n-2} g_m + \cdots + x^{n-2} g_0 = x^{n-2} G$$

$$\vdots$$

$$x^{m+1} g_m + \cdots + x g_0 = x G$$

$$x^{m} g_m + \cdots + x g_1 + g_0 = G$$

DEFINITION:

$$|F|_k = \sqrt[k]{|f_n|^k + \cdots + |f_0|^k} \qquad |F| = |F|_\infty$$

THEOREM: THERE EXIST POLYNOMIALS $A(x)$, $B(x) \in \mathbb{Z}[x]$ WITH $\deg(A) \leq m-1$, $\deg(B) \leq n-1$ SUCH THAT

$$R(F,G) = A(x)F(x) + B(x)G(x)$$

AND

$$|A|, |B| \leq |F|_2^m \, |G|_2^n.$$

FACT: $R(F,G) = 0 \iff GCD(F,G) \neq 1.$

FACT: LET η BE A ROOT OF F. THEN

$$|\eta| \leq 2 \max\left(1, \frac{|F|}{|f_n|}\right).$$

FACT: IF F_1 DIVIDES F THEN

$$|F_1| \leq \binom{n}{\lfloor n/2 \rfloor} |F|_2 .$$

LET α BE A RATIONAL APPROXIMATION OF η:

$$|\alpha - \eta| \leq 2^{-e}.$$

LET G BE OF DEGREE $n-1$ WITH

$$|G| \leq \binom{n}{\lfloor n/2 \rfloor} |F|_2$$

AND $G(\eta)=0$. THEN THERE EXIST NUMBERS $\varphi(n, |F|)$ AND $\gamma(n, |F|)$ SUCH THAT

$$|F(\alpha)|, |G(\alpha)| \leq 2^{-e+\varphi}$$

$$|A(\alpha)|, |B(\alpha)| \leq 2^{\gamma}$$

WE CHOOSE $e \geq \varphi + \gamma + 2$ AND SOLVE

$$|G(\alpha)| \leq 2^{-e+\varphi}, \; |G| \leq \binom{n}{\lfloor n/2 \rfloor} |F|_2$$

AS AN $\boxed{\text{INTEGER LINEAR PROGRAMMING}}$ PROBLEM IN THE COEFFICIENTS OF G.

THEN $R(F, G) = A(\alpha) F(\alpha) + B(\alpha) G(\alpha) = 0$

HENCE $\mathrm{GCD}(F, G) \neq 0$!

A. LENSTRA, H. LENSTRA AND L. LOVASZ USE p-ADIC VALUATIONS AND SOLVE THE ILP BY A BASIS CONSTRUCTION FOR INTEGRAL LATTICES.

MY VALUATION IS THE TOTAL DEGREE OF THE COEFFICIENTS WHICH MAKES THE ILP INTO A SYSTEM OF LINEAR EQU'S.

[GIVEN $f(x,y) \in \mathbb{Z}[x,y]$ THIS ALGORITHM RETURNS AN IRREDUCIBLE FACTOR $g(x,y)$:]

I: PREPROCESS f! [MAKES f MONIC IN x WITH $f(0,x)$ SQUAREFREE.]

F: FACTOR $f(0,x)$. LET $h(x)$ BE AN IRREDUCIBLE FACTOR, β ONE OF ITS ROOTS.

N: $n = \deg_x(f)$, $d = \deg_y(f)$, $m = \deg(h)$.
$K \leftarrow \lceil d(2n-1)/m \rceil$.
CONSTRUCT $\alpha_K(\beta) = \beta + \hat{\alpha}_1(\beta)y \cdots \hat{\alpha}_K(\beta)y^K$
SUCH THAT $f(y, \alpha_K(\beta)) \equiv 0 \mod y^{K+1}$
BY NEWTON ITERATION.

L: FOR $I \leftarrow m, \ldots, n-1$ DO
SOLVE

$$\alpha_K(\beta)^I + \sum_{i=0}^{I-1} u_i(y)\,\alpha_K(\beta)^i \equiv 0 \mod y^{K+1}$$

FOR $\deg(u_i) = d$. THE FIRST SOLUTION DETERMINES A FACTOR $x^I + u_{I-1}(y)x^{I-1} + \cdots + u_0(y)$
[$(K+1)m$ EQU'S, $(d+1)I$ UNKNOWNS.]

Appendix A

```
     Vaxima 1.50
   Sat Mar  6 01:41:14 1982

(c2) /* Sample run of algorithm 3.1: */

/* Inhibit garbage collection message */

gcprint:false$

(c3) /* The following bi-variate polynomial is squarefree and
monic in x as well as 0 is already a useable evaluation
point for y. Therefore, step (I) is not needed. */

f:x^6+x^5+(2*y+4)*x^4+(y+3)*x^3+(y^2+3*y+5)*x^2+(2-y)*x-y^2+y+2;
       2   2               2    4               3                              6
(d3) x  (y  + 3 y + 5) - y  + x  (2 y + 4) + x  (y + 3) + y + x (2 - y) + x

                                                                             5
                                                                         + x  + 2

(c4) /* Step (F): */

factor(subst(0,y,f));
                                2          2          2
(d4)                        (x  + 1) (x  + 2) (x  + x + 1)

(c5) /* We choose b=%i, the imaginary unit. */

/* Step (N): */

/* Step (N1): */

g[0]:x-%i;
(d5)                               x - %i

(c6) h[0]:quotient(subst(0,y,f),g[0]);
      5              4              3                2
(d6) x  + (%i + 1) x  + (%i + 3) x  + (3 %i + 2) x  + (2 %i + 2) x + 2 %i

(c7) /* Step (N2). We actually choose K smaller than described,
though this does not influence the outcome of latter steps */

K:5;
(d7)                                   5

(c8) /* Precompute inverse of f'(0,%i)=h[0](%i). */
```

```
r:ratsimp(1/subst(%i,x,h[0]));
```

(d8) $$-\frac{1}{2}$$

```
(c9) /* Computation of g[j] and h[j] using B[j]: */

for j:1 thru K do (
    display(f[j]:ratcoeff(f,y,j)),
    /* Step (N2): */
    if j=0
       then display(B[j]:f[j])
       else display(B[j]:ratsimp(f[j]-sum(g[s]*h[j-s],s,1,j-1))),
    display(g[j]:ratsimp(subst(%i,x,B[j])*r)),
    display(h[j]:quotient(B[j]-h[0]*g[j],g[0]))
    );
```

$$f_1 = 2\,x^4 + x^3 + 3\,x^2 - x + 1$$

$$b_1 = 2\,x^4 + x^3 + 3\,x^2 - x + 1$$

$$g_1 = \%i$$

$$h_1 = - \%i\,x^4 + (4 - \%i)\,x^3 + (\%i + 3)\,x^2 + (\%i + 5)\,x + 3\,\%i$$

$$f_2 = x^2 - 1$$

$$b_2 = - x^4 + (- 4\,\%i - 1)\,x^3 + (2 - 3\,\%i)\,x^2 + (1 - 5\,\%i)\,x + 2$$

$$g_2 = - \frac{5\,\%i}{2}$$

$$h_2 = \frac{5\,\%i\,x^4 + (5\,\%i - 12)\,x^3 + (- 5\,\%i - 12)\,x^2 + (- 8\,\%i - 6)\,x - 6\,\%i}{2}$$

$$f_3 = 0$$

$$b_3 = - \frac{- 10 x^4 + (- 32 \%i - 10) x^3 + (10 - 27 \%i) x^2 + (13 - 31 \%i) x + 21}{2}$$

$$g_3 = \frac{25 \%i}{2}$$

$$h_3 =$$

$$- \frac{25 \%i x^4 + (25 \%i - 60) x^3 + (- 17 \%i - 60) x^2 + (- 37 \%i - 48) x - 29 \%i}{2}$$

$$f_4 = 0$$

$$b_4 =$$

$$\frac{- 125 x^4 + (- 380 \%i - 125) x^3 + (109 - 330 \%i) x^2 + (164 - 376 \%i) x + 238}{4}$$

$$g_4 = - \frac{619 \%i}{8}$$

$$h_4 = (619 \%i x^4 + (619 \%i - 1488) x^3 + (- 391 \%i - 1488) x^2$$

$$+ (- 910 \%i - 1248) x - 762 \%i)/8$$

$$f_5 = 0$$

$$b_5 = - (- 1738 x^4 + (- 5164 \%i - 1738) x^3 + (1430 - 4545 \%i) x^2$$

$$+ (2299 - 5123 \%i) x + 3209)/8$$

$$g_5 = \frac{4291 \%i}{8}$$

$$h_5 = -(4291\,\%i\,x^4 + (4291\,\%i - 10320)\,x^3 + (-2611\,\%i - 10320)\,x^2 + (-6283\,\%i - 8832)\,x - 5373\,\%i)/8$$

(d9) done

(c10) /* Assign K-th order approximation of root of f. */

```
a[K]:%i+sum(-g[j]*y^j,j,1,K);
```

(d10) $$-\frac{4291\,\%i\,y^5}{8} + \frac{619\,\%i\,y^4}{8} - \frac{25\,\%i\,y^3}{2} + \frac{5\,\%i\,y^2}{2} - \%i\,y + \%i$$

(c11) /* This command verifies our approximation as it was proven. */

```
remainder(subst(a[K],x,f),y^(K+2));
```

(d11) $$\frac{63729\,\%i\,y^6}{8}$$

(c12) /* Compute powers of a[K] mod y^(K+1). */

```
asquare[K]:remainder(a[K]^2,y^(K+1));
```

(d12) $$1290\,y^5 - 186\,y^4 + 30\,y^3 - 6\,y^2 + 2\,y - 1$$

(c13)
```
acube[K]:remainder(a[K]*asquare[K],y^(K+1));
```

(d13) $$\frac{18537\,\%i\,y^5 - 2667\,\%i\,y^4 + 428\,\%i\,y^3 - 84\,\%i\,y^2 + 24\,\%i\,y - 8\,\%i}{8}$$

(c14)
```
afourth[K]:remainder(a[K]*acube[K],y^(K+1));
```

(d14) $$-3684\,y^5 + 528\,y^4 - 84\,y^3 + 16\,y^2 - 4\,y + 1$$

(c15) /* Set up undetermined polynomials of possible factor. */

```
u[0]:w0+v0*y+u0*y^2;
```

(d15) $$u0\,y^2 + v0\,y + w0$$

(c16)
```
u[1]:w1+v1*y+u1*y^2;
```

(d16) $$u1\,y^2 + v1\,y + w1$$

(c17)
```
u[2]:w2+v2*y+u2*y^2;
```

(d17) $$u2\, y^2 + v2\, y + w2$$

```
(c18) u[3]:w3+v3*y+u3*y^2;
```

(d18) $$u3\, y^2 + v3\, y + w3$$

```
(c19) /* Compute equation (3.1) for I=2. */

L2:remainder(asquare[K]+u[1]*a[K]+u[0],y^(K+1));
```

(d19) $$- ((4291\ \%i\ w1 - 619\ \%i\ v1 + 100\ \%i\ u1 - 10320)\ y^5$$

$$+ (- 619\ \%i\ w1 + 100\ \%i\ v1 - 20\ \%i\ u1 + 1488)\ y^4$$

$$+ (100\ \%i\ w1 - 20\ \%i\ v1 + 8\ \%i\ u1 - 240)\ y^3$$

$$+ (- 20\ \%i\ w1 + 8\ \%i\ v1 - 8\ \%i\ u1 - 8\ u0 + 48)\ y^2$$

$$+ (8\ \%i\ w1 - 8\ \%i\ v1 - 8\ v0 - 16)\ y - 8\ \%i\ w1 - 8\ w0 + 8)/8$$

```
(c20) t2:[]$

(c21) /* Retrieve linear equations for the coefficients. */

for i:0 thru K do
    for j:0 thru 1 do (
        display(s2[i,j]:ratcoeff(ratcoeff(L2,y,i),%i,j)),
        t2:cons(s2[i,j],t2)
        );
```

$$s2_{0,\,0} = w0 - 1$$

$$s2_{0,\,1} = w1$$

$$s2_{1,\,0} = v0 + 2$$

$$s2_{1,\,1} = v1 - w1$$

$$s2_{2,\,0} = u0 - 6$$

$$s2_{2,\,1} = \frac{5\ w1 - 2\ v1 + 2\ u1}{2}$$

$$s2_{3,0} = 30$$

$$s2_{3,1} = -\frac{25\ w1 - 5\ v1 + 2\ u1}{2}$$

$$s2_{4,0} = -186$$

$$s2_{4,1} = \frac{619\ w1 - 100\ v1 + 20\ u1}{8}$$

$$s2_{5,0} = 1290$$

$$s2_{5,1} = -\frac{4291\ w1 - 619\ v1 + 100\ u1}{8}$$

(d21) done

(c22) /* Try to solve the system. */

```
errcatch(linsolve(t2,
                  [w0,v0,u0,w1,v1,u1]));
Dependent equations eliminated:  (1 5 7)
Inconsistent equations:  (2 4 6)
```

(d22) []

(c23) /* Compute equation (3.1) for I=3. */

```
L3:remainder(acube[K]+u[2]*asquare[K]+u[1]*a[K]+u[0],y^(K+1));
```

(d23)
$$\big((10320\ w2 - 4291\ \%i\ w1 - 1488\ v2 + 619\ \%i\ v1 + 240\ u2 - 100\ \%i\ u1 + 18537\ \%i)\ y^5 + (-1488\ w2 + 619\ \%i\ w1 + 240\ v2 - 100\ \%i\ v1 - 48\ u2 + 20\ \%i\ u1 - 2667\ \%i)\ y^4 + (240\ w2 - 100\ \%i\ w1 - 48\ v2 + 20\ \%i\ v1 + 16\ u2 - 8\ \%i\ u1 + 428\ \%i)\ y^3 + (-48\ w2 + 20\ \%i\ w1 + 16\ v2 - 8\ \%i\ v1 - 8\ u2 + 8\ \%i\ u1 + 8\ u0 - 84\ \%i)\ y^2 + (16\ w2 - 8\ \%i\ w1 - 8\ v2 + 8\ \%i\ v1 + 8\ v0 + 24\ \%i)\ y - 8\ w2 + 8\ \%i\ w1 + 8\ w0 - 8\ \%i\big)/8$$

```
(c24) t3:[]$

(c25) /* Retrieve linear equations for the coefficients. */

for i:0 thru K do
    for j:0 thru 1 do (
        display(s3[i,j]:ratcoeff(ratcoeff(L3,y,i),%i,j)),
        t3:cons(s3[i,j],t3)
        );
```

$$s3_{0, 0} = w0 - w2$$

$$s3_{0, 1} = w1 - 1$$

$$s3_{1, 0} = 2\ w2 - v2 + v0$$

$$s3_{1, 1} = - w1 + v1 + 3$$

$$s3_{2, 0} = - 6\ w2 + 2\ v2 - u2 + u0$$

$$s3_{2, 1} = \frac{5\ w1 - 2\ v1 + 2\ u1 - 21}{2}$$

$$s3_{3, 0} = 30\ w2 - 6\ v2 + 2\ u2$$

$$s3_{3, 1} = - \frac{25\ w1 - 5\ v1 + 2\ u1 - 107}{2}$$

$$s3_{4, 0} = - 186\ w2 + 30\ v2 - 6\ u2$$

$$s3_{4, 1} = \frac{619\ w1 - 100\ v1 + 20\ u1 - 2667}{8}$$

$$s3_{5, 0} = 1290\ w2 - 186\ v2 + 30\ u2$$

$$s3_{5, 1} = - \frac{4291\ w1 - 619\ v1 + 100\ u1 - 18537}{8}$$

(d25) done

```
(c26) /* Try to solve the system. */

errcatch(linsolve(t3,
                        [w0,v0,u0,w1,v1,u1,w2,v2,u2]));
Inconsistent equations:  (5 7 1)

(d26)                                     []

(c27) /* Compute equation (3.1) for I=4. */

L4:remainder(afourth[K]+u[3]*acube[K]+u[2]*asquare[K]+u[1]*a[K]+u[0],y^(K+1));
(d27) ((18537 %i w3 + 10320 w2 - 4291 %i w1 - 2667 %i v3 - 1488 v2 + 619 %i v1

                                                  5
 + 428 %i u3 + 240 u2 - 100 %i u1 - 29472) y

 + (- 2667 %i w3 - 1488 w2 + 619 %i w1 + 428 %i v3 + 240 v2 - 100 %i v1

                                              4
 - 84 %i u3 - 48 u2 + 20 %i u1 + 4224) y

 + (428 %i w3 + 240 w2 - 100 %i w1 - 84 %i v3 - 48 v2 + 20 %i v1 + 24 %i u3

                                 3
 + 16 u2 - 8 %i u1 - 672) y  + (- 84 %i w3 - 48 w2 + 20 %i w1 + 24 %i v3

                                                                2
 + 16 v2 - 8 %i v1 - 8 %i u3 - 8 u2 + 8 %i u1 + 8 u0 + 128) y

 + (24 %i w3 + 16 w2 - 8 %i w1 - 8 %i v3 - 8 v2 + 8 %i v1 + 8 v0 - 32) y

 - 8 %i w3 - 8 w2 + 8 %i w1 + 8 w0 + 8)/8

(c28) t4:[]$

(c29) /* Retrieve linear equations for the coefficients. */

for i:0 thru K do
    for j:0 thru 1 do (
        display(s4[i,j]:ratcoeff(ratcoeff(L4,y,i),%i,j)),
        t4:cons(s4[i,j],t4)
        );
                                s4     = - w2 + w0 + 1
                                  0, 0

                                  s4     = w1 - w3
                                    0, 1

                              s4     = 2 w2 - v2 + v0 - 4
                                1, 0
```

$$s4_{1,\,1} = 3\ w3 - w1 - v3 + v1$$

$$s4_{2,\,0} = - 6\ w2 + 2\ v2 - u2 + u0 + 16$$

$$s4_{2,\,1} = - \frac{21\ w3 - 5\ w1 - 6\ v3 + 2\ v1 + 2\ u3 - 2\ u1}{2}$$

$$s4_{3,\,0} = 30\ w2 - 6\ v2 + 2\ u2 - 84$$

$$s4_{3,\,1} = \frac{107\ w3 - 25\ w1 - 21\ v3 + 5\ v1 + 6\ u3 - 2\ u1}{2}$$

$$s4_{4,\,0} = - 186\ w2 + 30\ v2 - 6\ u2 + 528$$

$$s4_{4,\,1} = - \frac{2667\ w3 - 619\ w1 - 428\ v3 + 100\ v1 + 84\ u3 - 20\ u1}{8}$$

$$s4_{5,\,0} = 1290\ w2 - 186\ v2 + 30\ u2 - 3684$$

$$s4_{5,\,1} = \frac{18537\ w3 - 4291\ w1 - 2667\ v3 + 619\ v1 + 428\ u3 - 100\ u1}{8}$$

```
(d29)                                done

(c30) /* Try to solve the system. */

linsolve(t4,
        [w0,v0,u0,w1,v1,u1,w2,v2,u2,w3,v3,u3]);
Solution

(e30)                              w1 = 0

(e31)                              v1 = 0

(e32)                              w0 = 2

(e33)                              u2 = 0

(e34)                              w2 = 3

(e35)                              v2 = 1
```

```
(e36)                                  u0 = 0

(e37)                                 v0 = - 1

(e38)                                  u1 = 0

(e39)                                  w3 = 0

(e40)                                  v3 = 0

(e41)                                  u3 = 0
(d41)    [e30, e31, e32, e33, e34, e35, e36, e37, e38, e39, e40, e41]

(c42) /* Substitute solution into factor. */

g:ev(x^4+u[3]*x^3+u[2]*x^2+u[1]*x+u[0],%);
                               2            4
(d42)                         x  (y + 3) - y + x  + 2

(c43) /* Test whether it divides f as was proven. */

remainder(f,g);
(d43)                                    0

(c44) /* Finally we demonstrate what happens if K is too small (4). */

a[4]:remainder(a[K],y^5);
                       4          3           2
              619 %i y  - 100 %i y  + 20 %i y  - 8 %i y + 8 %i
(d44)         ------------------------------------------------
                                     8

(c45) asquare[4]:remainder(asquare[K],y^5);
                              4        3       2
(d45)                  - 186 y  + 30 y  - 6 y  + 2 y - 1

(c46) acube[4]:remainder(acube[K],y^5);
                        4          3          2
              2667 %i y  - 428 %i y  + 84 %i y  - 24 %i y + 8 %i
(d46)       - --------------------------------------------------
                                     8

(c47) afourth[4]:remainder(afourth[K],y^5);
                            4        3        2
(d47)                  528 y  - 84 y  + 16 y  - 4 y + 1

(c48) /* Compute equation (3.1) for I=4. */

L4:remainder(afourth[4]+u[3]*acube[4]+u[2]*asquare[4]+u[1]*a[4]+u[0],y^5);
```

```
(d48) - ((2667 %i w3 + 1488 w2 - 619 %i w1 - 428 %i v3 - 240 v2 + 100 %i v1

                                                  4
 + 84 %i u3 + 48 u2 - 20 %i u1 - 4224) y

 + (- 428 %i w3 - 240 w2 + 100 %i w1 + 84 %i v3 + 48 v2 - 20 %i v1 - 24 %i u3

                             3
 - 16 u2 + 8 %i u1 + 672) y   + (84 %i w3 + 48 w2 - 20 %i w1 - 24 %i v3 - 16 v2

                                                               2
 + 8 %i v1 + 8 %i u3 + 8 u2 - 8 %i u1 - 8 u0 - 128) y

 + (- 24 %i w3 - 16 w2 + 8 %i w1 + 8 %i v3 + 8 v2 - 8 %i v1 - 8 v0 + 32) y

 + 8 %i w3 + 8 w2 - 8 %i w1 - 8 w0 - 8)/8

(c49) t4:[]$

(c50) /* Retrieve linear equations for the coefficients. */

for i:0 thru 4 do
    for j:0 thru 1 do (
        display(s4[i,j]:ratcoeff(ratcoeff(L4,y,i),%i,j)),
        t4:cons(s4[i,j],t4)
        );
                              s4     = - w2 + w0 + 1
                                0, 0

                                 s4     = w1 - w3
                                   0, 1

                            s4     = 2 w2 - v2 + v0 - 4
                              1, 0

                           s4     = 3 w3 - w1 - v3 + v1
                             1, 1

                       s4     = - 6 w2 + 2 v2 - u2 + u0 + 16
                         2, 0

                    21 w3 - 5 w1 - 6 v3 + 2 v1 + 2 u3 - 2 u1
          s4     = - ----------------------------------------
            2, 1                        2

                         s4     = 30 w2 - 6 v2 + 2 u2 - 84
                           3, 0

                   107 w3 - 25 w1 - 21 v3 + 5 v1 + 6 u3 - 2 u1
         s4     =  -------------------------------------------
           3, 1                        2

                      s4     = - 186 w2 + 30 v2 - 6 u2 + 528
                        4, 0
```

$$s4_{4, 1} = - \frac{2667\, w3 - 619\, w1 - 428\, v3 + 100\, v1 + 84\, u3 - 20\, u1}{8}$$

(d50) done

(c51) /* Try to solve the system. */

```
linsolve(t4,
        [w0,v0,u0,w1,v1,u1,w2,v2,u2,w3,v3,u3]);
```

Solution

(e51) $$w1 = \frac{u3}{9}$$

(e52) $$v1 = \frac{2\, u3}{3}$$

(e53) $$w0 = \frac{v2 + 15}{8}$$

(e54) $$u2 = \frac{9\, v2 - 9}{8}$$

(e55) $$w2 = \frac{v2 + 23}{8}$$

(e56) $$u0 = - \frac{v2 - 1}{8}$$

(e57) $$v0 = \frac{3\, v2 - 7}{4}$$

(e58) $$u1 = - \frac{u3}{9}$$

(e59) $$w3 = \frac{u3}{9}$$

(e60) $$v3 = \frac{8\, u3}{9}$$

```
(d60)          [e51, e52, e53, e54, e55, e56, e57, e58, e59, e60]

(c61) /* We now specialize u3=0 and v2=0 in the above solution. */

g:ev(ev(x^4+u[3]*x^3+u[2]*x^2+u[1]*x+u[0],%),u3=0,v2=0);
                               2            2
                              y      2 23   9 y     7 y    4   15
(d61)                         -- + x  (-- - ----) - --- + x  + --
                              8        8     8       4         8

(c62) /* g does not divide f, however */

resultant(f,g,x);

/user/vaxima/rat/result being loaded.
[fasl /user/vaxima/rat/result.o]
               10      6        5         4         3         2
(d62)  5184 y    (81 y  + 324 y  - 135 y  - 806 y  + 865 y  - 98 y + 169)

(c63) /* which is divisible by y^4 explaining the problem. */
```