1967	E. BERLEKAMP	POLYNOMIAL-TIME ALGORITHM FOR COEFFICIENTS IN FINITE FIELD
1969	H. ZASSENHAUS	HENSEL TECHNIQUE FOR UNIVARIATE CASE
1971	D. MUSSER	HENSEL TECHNIQUE FOR MULTIVARIATE CASE
1975	P. WANG, P. WEINBERGER	HENSEL TECHNIQUE FOR ALGEBRAIC EXTENSION CASE
1976	M. RABIN	PROBABILISTIC ALGORITHM FOR UNIVARIATE POLYNOMIALS OVER LARGE FINITE FIELDS
1980	R. ZIPPEL	PROBABILISTIC ALGORITHM FOR SPARSE MULTIVARIATE POLYNOMIALS
1981	D. CANTOR	SUCCINCT CERTIFICATES
	P. WEINBERGER	POLYNOMIAL TIME IRREDUCIBILITY TEST (UNDER GRH)

1982 A.K. LENSTRA, H.W. LENSTRA, L. LOVÁSZ — CAN FACTOR UNIVARIATE POLYNOMIAL IN TIME

$$O\left(\deg(f)^{12} + \deg(f)^{9} \log(\text{norm}(f))^{3}\right)$$

MY RESULTS:

THERE IS AN ALGORITHM WHICH REDUCES FACTORIZATION OF A MULTIVARIATE POLYNOMIAL WITH A FIXED NUMBER OF VARIABLES TO FACTORIZATION OF UNIVARIATE POLYNOMIALS IN TIME POLYNOMIAL IN THE TOTAL AND THE SIZE OF THE COEFFICIENTS.

PREPROCESSING OF INPUT:

1. COMPUTE CONTENT AND PRIMITIVE PART

$$\mathrm{cont}_x(f(u,v,x)) = \mathrm{GCD}\left(\mathrm{coeff}_x(f(u,v,x))\right)$$

$$\mathrm{PP}_x(f) = \frac{1}{\mathrm{cont}_x(f)}\, f = \bar{f}$$

2. SQUAREFREE DECOMPOSE PRIM. PART $\bar{f}$

TRICK:

IF $\bar{f} = g h^2$ THEN $\frac{\partial \bar{f}}{\partial x} = h\left(\frac{\partial g}{\partial x} h + 2 g \frac{\partial h}{\partial x}\right)$

HENCE $\mathrm{GCD}\left(\bar{f}, \frac{\partial f}{\partial x}\right)$ IS NON-TRIVIAL

$\bar{f} = \prod_{i=1}^{r} g_i^{\,i}$ WITH g_i SQUAREFREE

AND $\mathrm{GCD}(g_i, g_j) = 1,\ i \neq j$

POLYNOMIAL GCD's CAN BE DONE IN TIME POLYNOMIAL IN f. (G. COLLINS, W.S. BROWN)

3. MAKE SQUAREFREE FACTOR g MONIC

LET $c(u,v) = ldcf_x(g(u,v,x))$, THE LEADING COEFFICIENT OF g IN x.

$$h(u,v,x) = c(u,v)^{deg(g)-1}\, g\left(u,v,\frac{x}{c(u,v)}\right)$$

h IS MONIC (I.E. $ldcf_x(h) = 1$) AND IF $h = h_1 \cdots h_m$ IS A FACTORIZATION OF h THEN

$$g(u,v,x) = \prod_{i=1}^{m} pp_x(h_i(u,v,c(u,v)\,x))$$

IS ONE FOR g.

4. MAKE $(u,v) = (0,0)$ A REGULAR POINT

FIND INTEGERS a, b SUCH THAT

$$\bar{h}(\bar{u},\bar{v},x) = h(\bar{u}+a, \bar{v}+b, x)$$

IS SQUARE FREE AT $\bar{u} = 0, \bar{v} = 0$.

LEMMA: LET $d = \max(deg_u(h), deg_v(h))$. THEN THERE EXIST INTEGERS a, b WITH $0 \leq a, b \leq 2d^2 - d$ SUCH THAT $h(a,b,x)$ REMAINS SQUAREFREE.

[GIVEN $f(x,y) \in \mathbb{Z}[x,y]$ THIS ALGORITHM RETURNS AN IRREDUCIBLE FACTOR $g(x,y)$:]

I: PREPROCESS f ! [MAKES f MONIC IN x WITH $f(0,x)$ SQUAREFREE.]

F: FACTOR $f(0,x)$. LET $h(x)$ BE AN IRREDUCIBLE FACTOR, β ONE OF ITS ROOTS.

N: $n = \deg_x(f)$, $d = \deg_y(f)$, $m = \deg(h)$.
$K \leftarrow \lceil d(2n-1)/m \rceil$.
CONSTRUCT $\alpha_K(\beta) = \beta + \hat{\alpha}_1(\beta)y \cdots \hat{\alpha}_K(\beta)y^K$
SUCH THAT $f(y, \alpha_K(\beta)) \equiv 0 \bmod y^{K+1}$
BY NEWTON ITERATION.

L: FOR $I \leftarrow m, \ldots, n-1$ DO
SOLVE

$$\alpha_K(\beta)^I + \sum_{i=0}^{I-1} u_i(y)\,\alpha_K(\beta)^i \equiv 0 \bmod y^{K+1}$$

FOR $\deg(u_i) = d$. THE FIRST SOLUTION DETERMINES A FACTOR $x^I + u_{I-1}(y)x^{I-1} + \cdots + u_0(y)$
[$(K+1)m$ EQU'S, $(d+1)I$ UNKNOWNS.]

```
% /vc/mac/svaxima

     Vaxima 1.00
  Sun Nov 29 14:51:18 1981

(c1) showtime:true$

Time= 0 msec.

(c2) gcprint:false$

Time= 0 msec.

(c3) batch(red);


(c4) f:x^6+x^5+(2*y+4)*x^4+(y+3)*x^3+(y^2+3*y+5)*x^2+(2 y)*x-y^2+y+2;
Time= 183 msec.
```

$$(d4)\quad x^6 + x^5 + (2y+4)x^4 + (y+3)x^3 + (y^2+3y+5)x^2 + (2y)x - y^2 + y + 2$$

```
(c5) factor(subst(0,y,f));
Time= 2233 msec.
```

$$(d5)\quad (x^2+1)(x^2+2)(x^2+x+1)$$

```
(c6) gfactor(subst(0,y,f));
[autoload /vc/mac/rat/result]
[fasl /vc/mac/rat/result.o]
Time= 3616 msec.
```

$$(d6)\quad (x-\%i)(x+\%i)(x^2+2)(x^2+x+1)$$

```
(c7) a[7]:-495741*%i*y^7/16+63729*%i*y^6/16-4291*%i*y^5/8+619*%i*y^4/8
     -25*%i*y^3/2+5*%i*y^2/2-%i*y+%i;
Time= 300 msec.
```

$$(d7)\quad -\frac{495741\ \%i\ y^7}{16} + \frac{63729\ \%i\ y^6}{16} - \frac{4291\ \%i\ y^5}{8} + \frac{619\ \%i\ y^4}{8} - \frac{25\ \%i\ y^3}{2} + \frac{5\ \%i\ y^2}{2} - \%i\ y + \%i$$

```
(c8) remainder(subst(a[7],x,f),y^9);
Time= 14700 msec.
```

$$(d8)\quad \frac{31900851\ \%i\ y^8}{64}$$

```
(c9) asquare[7]:remainder(a[7]^2,y^8);
Time= 533 msec.
                  7         6         5        4       3      2
(d9)    74550 y  - 9582 y  + 1290 y  - 186 y  + 30 y  - 6 y  + 2 y - 1

(c10) acube[7]:remainder(a[7]*asquare[7],y^8);
Time= 550 msec.
                   7               6              5             4           3
(d10) (2146743 %i y  - 275713 %i y  + 37074 %i y  - 5334 %i y  + 856 %i y

                                                          2
                                                 - 168 %i y  + 48 %i y - 16 %i)/16

(c11) afourth[7]:remainder(a[7]*acube[7],y^8);
Time= 600 msec.
                  7         6         5        4       3      2
(d11) - 214068 y  + 27456 y  - 3684 y  + 528 y  - 84 y  + 16 y  - 4 y + 1

(c12) u[0]:u00+u01*y+u02*y^2;
Time= 50 msec.
                                    2
(d12)                         u02 y  + u01 y + u00

(c13) u[1]:u10+u11*y+u12*y^2;
Time= 16 msec.
                                    2
(d13)                         u12 y  + u11 y + u10

(c14) u[2]:u20+u21*y+u22*y^2;
Time= 33 msec.
                                    2
(d14)                         u22 y  + u21 y + u20

(c15) u[3]:u30+u31*y+u32*y^2;
Time= 16 msec.
                                    2
(d15)                         u32 y  + u31 y + u30


(c16) S2:remainder(asquare[7]+u[1]*a[7]+u[0],y^8);
Time= 833 msec.
                                                                        7
(d16) - ((8582 %i u12 - 63729 %i u11 + 495741 %i u10 - 1192800) y

                                                                    6
 + (- 1238 %i u12 + 8582 %i u11 - 63729 %i u10 + 153312) y

                                                                5
 + (200 %i u12 - 1238 %i u11 + 8582 %i u10 - 20640) y

                                                            4
 + (- 40 %i u12 + 200 %i u11 - 1238 %i u10 + 2976) y

                                                        3
 + (16 %i u12 - 40 %i u11 + 200 %i u10 - 480) y

                                                    2
 + (- 16 %i u12 + 16 %i u11 - 40 %i u10 - 16 u02 + 96) y

 + (- 16 %i u11 + 16 %i u10 - 16 u01 - 32) y - 16 %i u10 - 16 u00 + 16)/16
```

```
(c17) for i:0 thru 7 do
    for j:0 thru 1 do display(s2[i,j]:ratcoeff(ratcoeff(S2,y,i),%i,j));
```

$$s2_{0,0} = u00 - 1$$

$$s2_{0,1} = u10$$

$$s2_{1,0} = u01 + 2$$

$$s2_{1,1} = u11 - u10$$

$$s2_{2,0} = u02 - 6$$

$$s2_{2,1} = \frac{2\,u12 - 2\,u11 + 5\,u10}{2}$$

$$s2_{3,0} = 30$$

$$s2_{3,1} = -\frac{2\,u12 - 5\,u11 + 25\,u10}{2}$$

$$s2_{4,0} = -186$$

$$s2_{4,1} = \frac{20\,u12 - 100\,u11 + 619\,u10}{8}$$

$$s2_{5,0} = 1290$$

$$s2_{5,1} = -\frac{100\,u12 - 619\,u11 + 4291\,u10}{8}$$

$$s2_{6,0} = -9582$$

$$s2_{6,1} = \frac{1238\,u12 - 8582\,u11 + 63729\,u10}{16}$$

$$s2_{7,0} = 74550$$

$$s2_{7,1} = -\frac{8582\,u12 - 63729\,u11 + 495741\,u10}{16}$$

```
Time= 16400 msec.
(d17)                                done

(c18) linsolve([s2[0,0],s2[0,1],s2[1,0],s2[1,1],s2[2,0],s2[2,1],s2[3,0],s2[3,1],
         s2[4,0],s2[4,1],s2[5,0],s2[5,1],s2[6,0],s2[6,1],s2[7,0],s2[7,1]],
       [u00,u01,u02,u10,u11,u12]);
Dependent equations eliminated:  (16 12 10 14 8)
Inconsistent equations:  (7 9 15 11 13)Time= 3083 msec. so far
```

```
(c19) batcon(16);


(c20) S3:remainder(acube[7]+u[2]*asquare[7]+u[1]*a[7]+u[0],y^8);
Time= 1533 msec.
(d20) ((20640 u22 - 153312 u21 + 1192800 u20 - 8582 %i u12 + 63729 %i u11

                                                                    7
 - 495741 %i u10 + 2146743 %i) y  + (- 2976 u22 + 20640 u21 - 153312 u20

                                                                    6
 + 1238 %i u12 - 8582 %i u11 + 63729 %i u10 - 275713 %i) y

 + (480 u22 - 2976 u21 + 20640 u20 - 200 %i u12 + 1238 %i u11 - 8582 %i u10

                    5
 + 37074 %i) y  + (- 96 u22 + 480 u21 - 2976 u20 + 40 %i u12 - 200 %i u11

                                  4
 + 1238 %i u10 - 5334 %i) y  + (32 u22 - 96 u21 + 480 u20 - 16 %i u12

                                                       3
 + 40 %i u11 - 200 %i u10 + 856 %i) y  + (- 16 u22 + 32 u21 - 96 u20

                                                                    2
 + 16 %i u12 - 16 %i u11 + 40 %i u10 + 16 u02 - 168 %i) y

 + (- 16 u21 + 32 u20 + 16 %i u11 - 16 %i u10 + 16 u01 + 48 %i) y - 16 u20

 + 16 %i u10 + 16 u00 - 16 %i)/16
```

```
(c21) for i:0 thru 7 do
    for j:0 thru 1 do display(s3[i,j]:ratcoeff(ratcoeff(S3,y,i),%i,j));
                              s3     = u00 - u20
                                0, 0

                               s3     = u10 - 1
                                 0, 1

                          s3     = - u21 + 2 u20 + u01
                            1, 0

                             s3     = u11 - u10 + 3
                               1, 1

                       s3     = - u22 + 2 u21 - 6 u20 + u02
                         2, 0

                                 2 u12 - 2 u11 + 5 u10 - 21
                        s3     = ---------------------------
                          2, 1               2

                          s3     = 2 u22 - 6 u21 + 30 u20
                            3, 0

                                 2 u12 - 5 u11 + 25 u10 - 107
                       s3     = - ----------------------------
                         3, 1               2

                          s3     = - 6 u22 + 30 u21 - 186 u20
                            4, 0

                                 20 u12 - 100 u11 + 619 u10 - 2667
                       s3     = -----------------------------------
                         4, 1                8

                           s3     = 30 u22 - 186 u21 + 1290 u20
                             5, 0

                                  100 u12 - 619 u11 + 4291 u10 - 18537
                       s3     = - ------------------------------------
                         5, 1                 8

                            s3     = - 186 u22 + 1290 u21 - 9582 u20
                              6, 0

                                  1238 u12 - 8582 u11 + 63729 u10 - 275713
                       s3     = ------------------------------------------
                         6, 1                  16

                            s3     = 1290 u22 - 9582 u21 + 74550 u20
                              7, 0

                                  8582 u12 - 63729 u11 + 495741 u10 - 2146743
                       s3     = - -------------------------------------------
                         7, 1                   16

Time= 24650 msec.
(d21)                                  done

(c22) linsolve([s3[0,0],s3[0,1],s3[1,0],s3[1,1],s3[2,0],s3[2,1],s3[3,0],s3[3,1],
         s3[4,0],s3[4,1],s3[5,0],s3[5,1],s3[6,0],s3[6,1],s3[7,0],s3[7,1]],
       [u00,u01,u02,u10,u11,u12,u20,u21,u22]);
Dependent equations eliminated:  (13 11)
Inconsistent equations:  (8 16 10 14 12)Time= 8516 msec. so far
```

```
(c23) batcon(19);

(c24) S4:remainder(afourth[7]+u[3]*acube[7]+u[2]*asquare[7]+u[1]*a[7]+u[0],y^8);
Time= 1966 msec.
(d24) ((37074 %i u32 - 275713 %i u31 + 2146743 %i u30 + 20640 u22 - 153312 u21

                                                                        7
+ 1192800 u20 - 8582 %i u12 + 63729 %i u11 - 495741 %i u10 - 3425088) y

+ (- 5334 %i u32 + 37074 %i u31 - 275713 %i u30 - 2976 u22 + 20640 u21

                                                                     6
- 153312 u20 + 1238 %i u12 - 8582 %i u11 + 63729 %i u10 + 439296) y

+ (856 %i u32 - 5334 %i u31 + 37074 %i u30 + 480 u22 - 2976 u21 + 20640 u20

                                                                  5
- 200 %i u12 + 1238 %i u11 - 8582 %i u10 - 58944) y

+ (- 168 %i u32 + 856 %i u31 - 5334 %i u30 - 96 u22 + 480 u21 - 2976 u20

                                                               4
+ 40 %i u12 - 200 %i u11 + 1238 %i u10 + 8448) y

+ (48 %i u32 - 168 %i u31 + 856 %i u30 + 32 u22 - 96 u21 + 480 u20 - 16 %i u12

                                                            3
+ 40 %i u11 - 200 %i u10 - 1344) y  + (- 16 %i u32 + 48 %i u31 - 168 %i u30

- 16 u22 + 32 u21 - 96 u20 + 16 %i u12 - 16 %i u11 + 40 %i u10 + 16 u02 + 256)

  2
y  + (- 16 %i u31 + 48 %i u30 - 16 u21 + 32 u20 + 16 %i u11 - 16 %i u10

+ 16 u01 - 64) y - 16 %i u30 - 16 u20 + 16 %i u10 + 16 u00 + 16)/16
```

```
(c25) for i:0 thru 7 do
    for j:0 thru 1 do display(s4[i,j]:ratcoeff(ratcoeff(S4,y,i),%i,j));
```

$$s4_{0,0} = -u20 + u00 + 1$$

$$s4_{0,1} = u10 - u30$$

$$s4_{1,0} = -u21 + 2\,u20 + u01 - 4$$

$$s4_{1,1} = -u31 + 3\,u30 + u11 - u10$$

$$s4_{2,0} = -u22 + 2\,u21 \quad 6\,u20 + u02 + 16$$

$$s4_{2,1} = -\frac{2\,u32 - 6\,u31 + 21\,u30 - 2\,u12 + 2\,u11 - 5\,u10}{2}$$

$$s4_{3,0} = 2\,u22 - 6\,u21 + 30\,u20 - 84$$

$$s4_{3,1} = \frac{6\,u32 - 21\,u31 + 107\,u30 - 2\,u12 + 5\,u11 - 25\,u10}{2}$$

$$s4_{4,0} = -6\,u22 + 30\,u21 - 186\,u20 + 528$$

$$s4_{4,1} = -\frac{84\,u32 - 428\,u31 + 2667\,u30 - 20\,u12 + 100\,u11 - 619\,u10}{8}$$

$$s4_{5,0} = 30\,u22 - 186\,u21 + 1290\,u20 - 3684$$

$$s4_{5,1} = \frac{428\,u32 - 2667\,u31 + 18537\,u30 - 100\,u12 + 619\,u11 - 4291\,u10}{8}$$

$$s4_{6,0} = -186\,u22 + 1290\,u21 - 9582\,u20 + 27456$$

$$s4_{6,1} = -\frac{5334\,u32 - 37074\,u31 + 275713\,u30 - 1238\,u12 + 8582\,u11 - 63729\,u10}{16}$$

$$s4_{7,0} = 1290\,u22 - 9582\,u21 + 74550\,u20 - 214068$$

$$s4_{7,1} =$$

$$\frac{37074\,u32 - 275713\,u31 + 2146743\,u30 - 8582\,u12 + 63729\,u11 - 495741\,u10}{16}$$

```
Time= 34033 msec.
(d25)                                done
```

```
(c26) linsolve([s4[0,0],s4[0,1],s4[1,0],s4[1,1],s4[2,0],s4[2,1],s4[3,0],s4[3,1],
          s4[4,0],s4[4,1],s4[5,0],s4[5,1],s4[6,0],s4[6,1],s4[7,0],s4[7,1]],
         [u00,u01,u02,u10,u11,u12,u20,u21,u22,u30,u31,u32]);
Dependent equations eliminated:  (14 16 13 15)
Solution

(e26)                           u10 = 0

(e27)                           u00 = 2

(e28)                           u11 = 0

(e29)                           u01 = - 1

(e30)                           u12 = 0

(e31)                           u30 = 0

(e32)                           u31 = 0

(e33)                           u32 = 0

(e34)                           u22 = 0

(e35)                           u20 = 3

(e36)                           u21 = 1

(e37)                           u02 = 0
Time= 13466 msec.
(d37)    [e26, e27, e28, e29, e30, e31, e32, e33, e34, e35, e36, e37]

Time= 53933 msec.
(d38)                          BATCH DONE

(c39) factor(f);

Time= 3683 msec.

                      2         2           4       2
(d39)           (y + x  + x + 1) (x  y - y + x  + 3 x  + 2)

(c40)
```

```
(c44) /* Finally we demonstrate what happens if K is too small (4). */

a[4]:remainder(a[K],y^5);
                    4              3            2
         619 %i y  - 100 %i y  + 20 %i y  - 8 %i y + 8 %i
(d44)    ---------------------------------------------------
                                  8

(c45) asquare[4]:remainder(asquare[K],y^5);
                         4        3      2
(d45)             - 196 y  + 30 y  - 6 y  + 2 y - 1

(c46) acube[4]:remainder(acube[K],y^5);
                      4              3           2
          2667 %i y  - 428 %i y  + 86 %i y  - 24 %i y + 8 %i
(d46)   - ---------------------------------------------------
                                   8

(c47) afourth[4]:remainder(afourth[K],y^5);
                        4       3       2
(d47)              528 y  - 96 y  + 16 y  - 4 y + 1

(c48) /* Compute equation (3.1) for I=4. */

L4:remainder(afourth[4]+u[3]*acube[4]+u[2]*asquare[4]+u[1]*a[4]+u[0],y^5);
(d48) - (((2667 %i w3 + 1488 w2 - 619 %i w1 - 428 %i v3 - 240 v2 + 100 %i v1

                                               4
 + 86 %i u3 + 68 u2 - 20 %i u1 - 4224) y

 + (- 428 %i w3 - 240 w2 + 100 %i w1 + 86 %i v3 + 68 v2 - 20 %i v1 - 24 %i u3

                                  3
 - 16 u2 + 8 %i u1 + 672) y  + (86 %i w3 + 68 w2 - 20 %i w1 - 24 %i v3 - 16 v2

                                                        2
 + 8 %i v1 + 8 %i u3 + 8 u2 - 8 %i u1 - 8 u0 - 128) y

 + (- 24 %i w3 - 16 w2 + 8 %i w1 + 8 %i v3 + 8 v2 - 8 %i v1 - 8 v0 + 32) y

 + 8 %i w3 + 8 w2 - 8 %i w1 - 8 w0 - 8)/8

(c49) t4:[]$
```

```
(c50) /* Retrieve linear equations for the coefficients. */

for i:0 thru 4 do
    for j:0 thru 1 do (
        display(s4[i,j]:ratcoeff(ratcoeff(L4,y,i),x[i,j])),
        t4:cons(s4[i,j],t4)
        )$
```

$$s4_{0,0} = -u2 + u0 + 1$$

$$s4_{0,1} = u1 - u3$$

$$s4_{1,0} = 2\,u2 - v2 + v0 - 4$$

$$s4_{1,1} = 3\,u3 - u1 - v3 + v1$$

$$s4_{2,0} = -6\,u2 + 2\,v2 - u2 + u0 + 16$$

$$s4_{2,1} = -\frac{21\,u3 - 5\,u1 - 6\,v3 + 2\,v1 + 2\,u3 - 2\,u1}{2}$$

$$s4_{3,0} = 30\,u2 - 6\,v2 + 2\,u2 - 84$$

$$s4_{3,1} = \frac{107\,u3 - 25\,u1 - 21\,v3 + 5\,v1 + 6\,u3 - 2\,u1}{2}$$

$$s4_{4,0} = -186\,u2 + 30\,v2 - 6\,u2 + 528$$

$$s4_{4,1} = -\frac{2667\,u3 - 619\,u1 - 428\,v3 + 100\,v1 + 84\,u3 - 20\,u1}{8}$$

```
(c50)                          done

(c51) /* Try to solve the system. */

linsolve(t4,
        [u0,v0,u0,u1,v1,u1,u2,v2,u2,u3,v3,u3])$
```

Solution

(o51) $$u1 = \frac{u3}{9}$$

(o52) $$v1 = \frac{2\,u3}{3}$$

(o53) $$u0 = \frac{v2 + 15}{8}$$

(o54) $$u2 = \frac{9\,v2 - 9}{8}$$

(o55) $$u2 = \frac{v2 + 23}{8}$$

(o56) $$u0 = -\frac{v2 - 1}{8}$$

(o57) $$v0 = \frac{3\,v2 - 7}{4}$$

(o58) $$u1 = \frac{u3}{9}$$

(o59) $$u3 = \frac{u3}{9}$$

(o60) $$v3 = \frac{8\,u3}{9}$$

(o60) [o51, o52, o53, o54, o55, o56, o57, o58, o59, o60]

(c61) /* We now specialize u3=0 and v2=0 in the above solution. */

ratsubst(ev(x^4+u[3]*x^3+u[2]*x^2+u[1]*x+u[0],X),u3=0,v2=0);

(o61) $$\frac{y^2}{8} + x^2\left(\frac{23}{8} - \frac{9\,y^2}{8}\right) - \frac{7\,y^2}{4} + x^4 + \frac{15}{8}$$

```
(c62) /* g does not divide f, however */

resultant(f,g,x);

/usr/maxima/rat/result being loaded.
[fasl /usr/maxima/rat/result.o]
                  10       6        5        4        3        2
(d62)  5184 y   (81 y  + 324 y  - 135 y  - 806 y  + 865 y  - 98 y + 169)

(c63) /* which is divisible by y^4 explaining the problem. */
```